Key Recovery Attacks against NTRU-based Somewhat Homomorphic Encryption Schemes

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Outline of the Talk

Outline

1. Quick overview of FHE
2. Previous work
3. Idea of our key recovery attacks
4. Details of attacks
5. Conclusion and future directions

Basic Definitions

We only assume bit-by-bit public-key encryption

Public Key Homomorphic Encryption Scheme

$\mathcal{E} = (\text{KeyGen}_{\mathcal{E}}, \text{Encrypt}_{\mathcal{E}}, \text{Decrypt}_{\mathcal{E}}, \text{Evaluate}_{\mathcal{E}})$, all run in poly. time.

$$\text{KeyGen}(\lambda) = (\text{sk}, \text{pk}), \text{Encrypt}(\text{pk}, m) = c$$

$$\text{Decrypt}(\text{sk}, c) = m', \text{Evaluate}(\text{pk}, C, (c_1, \ldots, c_r)) = c_e$$

Correct Homomorphic Decryption

$\mathcal{E}$ is correct for a given t-input circuit C if, $\forall (\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(\lambda)$, $\forall m_1, \ldots, m_t \in \{0, 1\}$, $\forall \overline{c} = (c_1, \ldots, c_t)$ with $c_i \leftarrow \text{Encrypt}_{\mathcal{E}}(\text{pk}, m_i)$

$$\text{Decrypt}(\text{sk}, \text{Evaluate}(\text{pk}, C, \overline{c})) = C(m_1, \ldots, m_t)$$

Homomorphic Encryption

$\mathcal{E}$ homomorphic for a class $\mathcal{C}$ of circuits: correct for all circuits $C \in \mathcal{C}$
$\mathcal{E}$ fully homomorphic encryption (FHE) scheme: correct for all boolean circuits
$\mathcal{E}$ somewhat homomorphic encryption (SHE) scheme: limited # of op.

Basic Definitions

Security Definitions

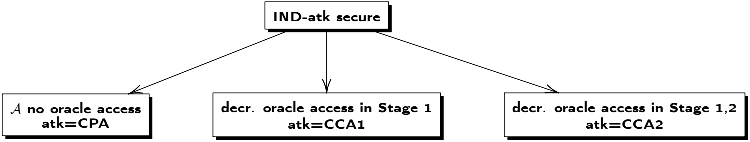
game between a challenger and an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$

- $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$
- $(m_0, m_1) \leftarrow \mathcal{A}_1^{(\cdot)}(\mathsf{pk})$ /* Stage 1 */
- $b \leftarrow \{0, 1\}$
- $c^* \leftarrow \mathsf{Encrypt}(m_b, \mathsf{pk})$
- $b' \leftarrow \mathcal{A}_2^{(\cdot)}(c^*)$ /* Stage 2 */

If $b = b'$: $\mathcal{A}$ wins game with

$$\mathrm{Adv}_{\mathcal{A},\mathcal{E},\lambda}^{\text{IND-atk}} = |\Pr(b = b') - 1/2|$$

Scheme IND-atk secure if no poly. time $\mathcal{A}$ wins with non-negl. adv.



Quick overview of FHE based on hardness assumptions

- 1978: Rivest et al [RAD78]: is it possible to perform arbitrary operations on encrypted ciphertexts? (privacy homomorphism / FHE)
- 2009: Gentry [Gen09b]: yes!

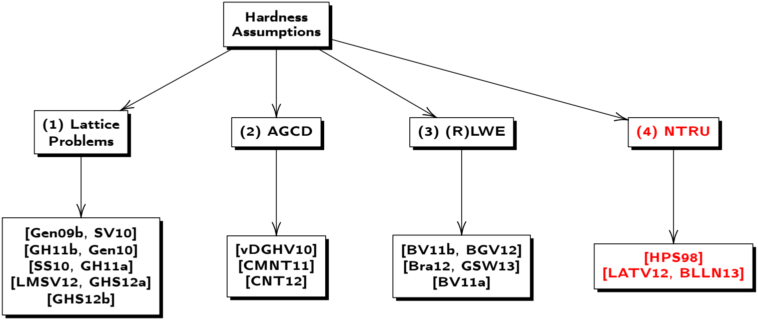


Figure : Hardness assumptions and relevant papers

Homomorphic Encryption and IND-CPA,CCA

- All known SHE and FHE schemes: IND-CPA secure
- No SHE and FHE scheme can be IND-CCA2
- With Gentry's approach, FHE scheme cannot be IND-CCA1 secure
- Open problem: investigate SHE schemes with IND-CCA1 security (Gentry [Gen09b])

Homomorphic Encryption and IND-CPA,CCA

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IND-CPA ✔	IND-CCA1 ?	IND-CCA2 ✗
Every scheme	Not known	No scheme

Key Recovery Attacks - Our Contribution

Our contribution

1. key recovery attack for SHE schemes in [LATV12, BLLN13]
2. SHE schemes in (4) above are not IND-CCA1 secure
3. conclusion: with results from [LMSV12, ZPS12, CT14], most existing SHE schemes (except [LMSV12]) suffer from key recovery attacks, so not IND-CCA1 secure

Key Recovery Attacks - The Idea

General line of work

- Premise: decryption oracle reveals one bit at a time or a polynomial in $Z_2[x]/(x^n + 1)$
- Idea: we submit to decryption oracle specifically-chosen 'ciphertexts' in order to get 1 bit of information for each coefficient of sk
- recover sk by gradually reducing (halving) the key space

Key Recovery Attack against SHE [LATV12]

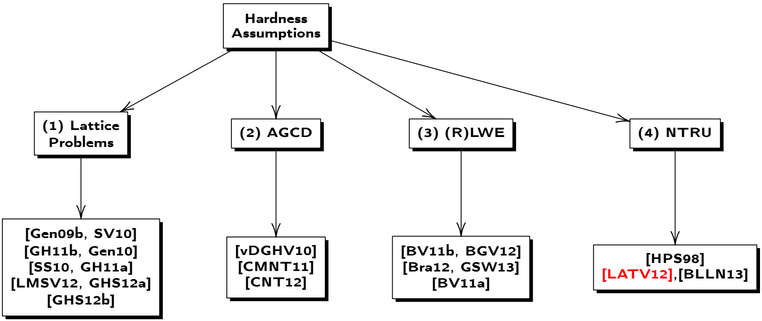


Figure : Hardness assumptions and relevant papers

Key Recovery Attack against [LATV12]

The [LATV12] SHE scheme (informal)

$\mathcal{M} = \mathbb{Z}_2$, $R := \mathbb{Z}[x]/(x^n + 1)$

KeyGen(λ) :

- $[\cdots]$
- sk $:= f \in R$

Encrypt(pk, m):

- sample $s, e \leftarrow \chi$
- output ciphertext $c := hs + 2e + m \in R_q$

Decrypt(sk, c):

- let $\mu = f \cdot c \in R_q$
- output $\mu' := \mu \bmod 2$

Comparison with [DGM15]

- attack exists in [DGM15], but require $6(t^2 + t) < q$ and $B^2 < \frac{q}{36t^2}$ (conditions not assumed in [LATV12])
- our attack: works for all parameters. More efficient than [DGM15]:

Our Attack	Attack from [DGM15]
$\lfloor \log_2 B \rfloor + n$	$n \cdot \lceil \log_2 B \rceil + n$

- n: power of 2; $B \ll q$ bound on coefficient of χ; $t \geq 2$ integer

Key Recovery Attack against [LATV12]

KeyGen(λ) :

- sk := $s(x) = s_0 + s_1 x + s_2 x^2 + \cdots + s_{n-1}x^{n-1} \in R_q$

Encrypt(pk, m):

- output ciphertext $c(x) \in R_q$

Decrypt(sk, $c(x)$):

- output $s(x) \cdot c(x) \in R_q \bmod 2$

Key recovery attack - The Idea

- determine the parity of each coefficient $s_i \in (-q/2, q/2]$
- determine $|s_i|$ by gradually halving the interval in which it lies
- at some point, $|s_i|$ belongs to some interval with at most two consecutive integers
- $|s_i|$ deduced by its known parity
- last step: query the oracle decryption at most n times in order to recover the sign of the coefficients s_i, for $i = 1, 2, \ldots, n-1$, relative to the (unknown) sign of s_0
- two possible candidate secret keys $s_1(x)$ and $s_2(x) = -s_1(x)$
- find whether $s(x) = s_1(x)$ or $s(x) = s_2(x)$ with extra oracle query

Key Recovery Attack against [LATV12] - Details

Preliminary Step

- submit to dec. oracle $c(x) = 1 \in R_q$
- oracle returns $D(c(x) = 1) = s(x) \bmod 2 = \sum_{i=0}^{n-1}(s_i \bmod 2)x^i$
- $\Rightarrow$ we learn parity of s_i, $i = 0, 1, \ldots, n-1$

Step 1

- submit to dec. oracle $c(x) = 2 \in R_q$
- oracle returns $D(c(x) = 2) = (2s(x) \in R_q) \bmod 2 = \sum_{i=0}^{n-1} [(2s_i \bmod q) \bmod 2]\, x^i$
- Now, $\forall i \in [0, n-1]$ we have

$$\frac{-q+1}{2} \le s_i \le \frac{q-1}{2}, \text{ and so } -q+1 \le 2s_i \le q-1 \qquad \text{(A)}$$

$\forall i$, two cases to distinguish:

Case A_1: $(2s_i \bmod q) \bmod 2 = 0$.
Then, condition (A) implies that
$\frac{-q+1}{2} \le 2s_i \le \frac{q-1}{2}$, i.e.
$\frac{-q+1}{4} \le s_i \le \frac{q-1}{4}$

$$-q+1 \le 4s_i \le q-1 \qquad \text{(A1)}$$

Case B_1: $(2s_i \bmod q) \bmod 2 = 1$.
Then, condition (A) implies that
$\frac{q-1}{2} + 1 \le 2|s_i| \le q-1$, i.e.
$\frac{q+1}{4} \le |s_i| \le \frac{q-1}{2}$

$$q+1 \le 4|s_i| \le 2q-2 \qquad \text{(B1)}$$

Key Recovery Attack against [LATV12] - Details

Step 2

- submit to dec. oracle $c(x) = 4 \in R_q$
- oracle returns $D(c(x) = 4) = [s(x) \cdot 4]_q \bmod 2 = \sum_{i=0}^{n-1} [[4s_i]_q \bmod 2]\, x^i$
- Now, $\forall i$, four cases to distinguish:

Case A_2**:** In Step 1 case A_1 held, and $[4s_i]_q \bmod 2 = 0$. Then, condition (A1) implies that $\frac{-q+1}{2} \leq 4s_i \leq \frac{q-1}{2}$, i.e. $\frac{-q+1}{8} \leq s_i \leq \frac{q-1}{8}$

$$-q+1 \leq 8s_i \leq q-1 \qquad \text{(A2)}$$

Case B_2**:** In Step 1 case A_1 held, and $[4s_i]_q \bmod 2 = 1$. Then, condition (A1) implies that $\frac{q-1}{2} + 1 \leq 4|s_i| \leq q-1$, i.e. $\frac{q+1}{8} \leq |s_i| \leq \frac{q-1}{4}$

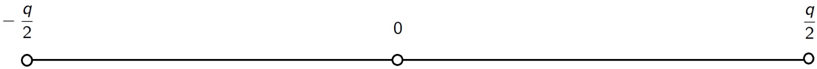
$$q+1 \leq 8|s_i| \leq 2q-2 \qquad \text{(B2)}$$

Case C_2**:** In Step 1 case B_1 held, and $[4s_i]_q \bmod 2 = 0$. Then, condition (B1) implies that $q+1+\frac{q-1}{2} \leq 4|s_i| \leq 2q-2$, i.e. $\frac{3q+1}{8} \leq |s_i| \leq \frac{q-1}{2}$

$$3q+1 \leq 8|s_i| \leq 4q-4 \qquad \text{(C2)}$$

Case D_2**:** In Step 1 case B_1 held, and $[4s_i]_q \bmod 2 = 1$. Then, condition (B1) implies that $q+1 \leq 4|s_i| \leq \frac{3q-1}{2}$, i.e. $\frac{q+1}{4} \leq |s_i| \leq \frac{3q-1}{8}$

$$2q+2 \leq 8|s_i| \leq 3q-1 \qquad \text{(D2)}$$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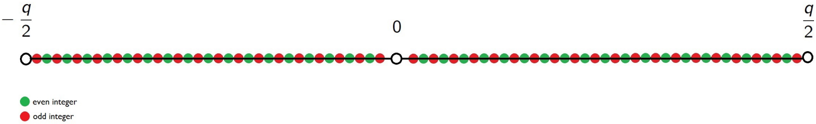
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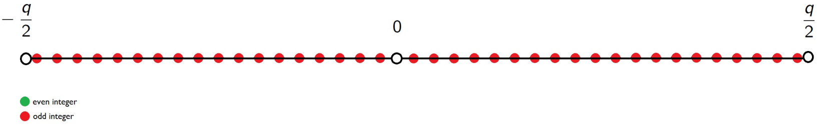

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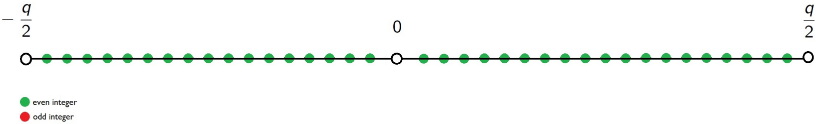
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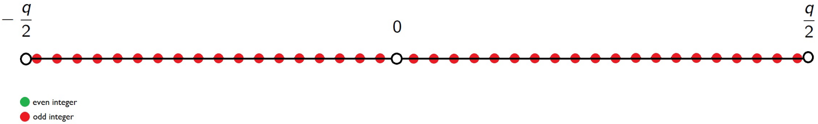

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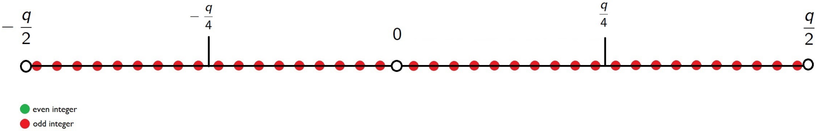

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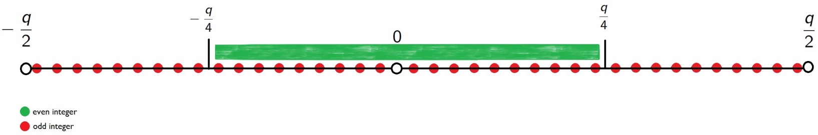

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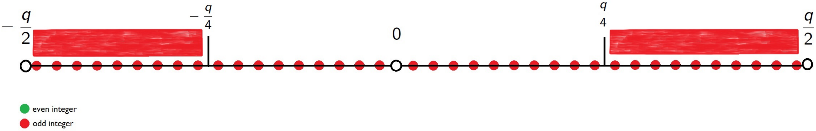

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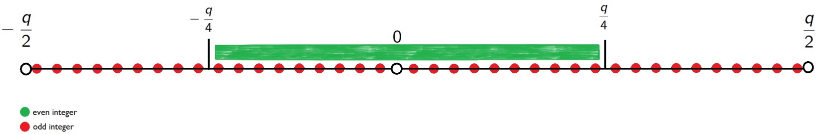

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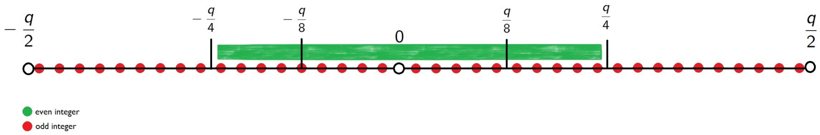
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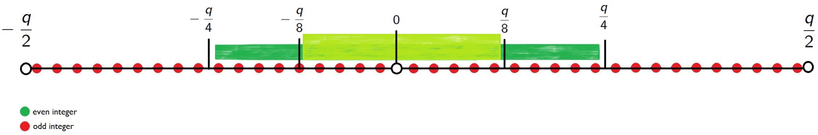

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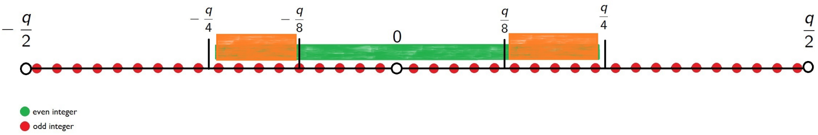

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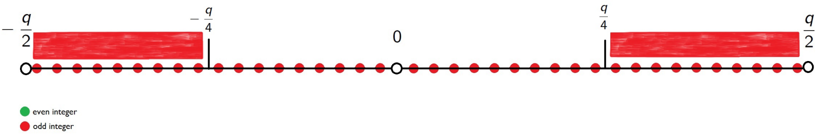

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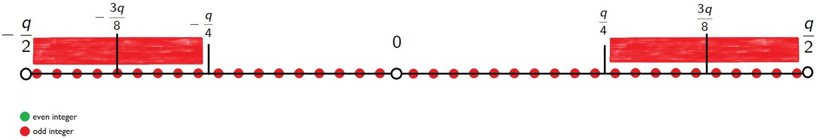
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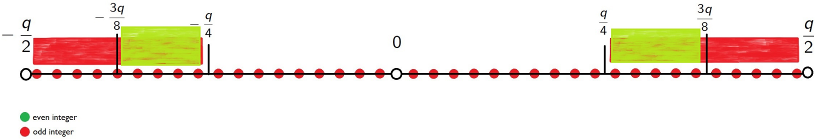

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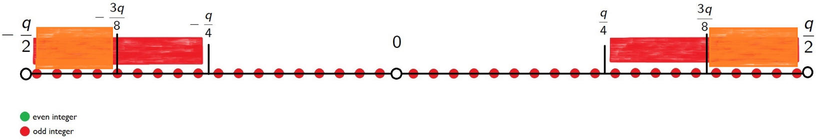
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Key Recovery Attack against [LATV12] - Details

Generalizing

- continue, and we find $s'_i := |s_i| \in [a_i, a_i + 1] \subseteq [0, \frac{q-1}{2}]$, for $i = 0, 1, \ldots, n-1$
- $|s_i|$ can assume at most only two (consecutive) values
- known parity $\Rightarrow$ determine $|s_i|$
- to achieve this we need $\lfloor \log_2 q \rfloor$ steps

Last step

- Left to find out whether $s_i \cdot s_j < 0$ or $s_i \cdot s_j > 0$, $\forall i, j$ with $s_i, s_j \neq 0$
- Let s_m be the first non-zero coefficient: we will obtain two possible candidates of sk, one with $s_m > 0$ and one with $s_m < 0$
- trivial oracle dec. query to determine which one is the correct sk
- omit details

Key Recovery Attack against SHE [BLLN13]

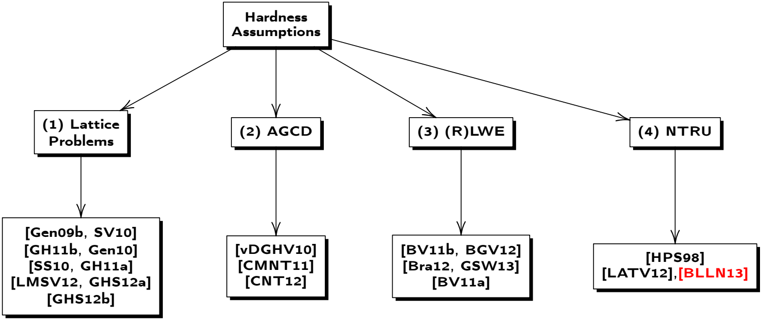

Figure : Hardness assumptions and relevant papers

Key Recovery Attack against [BLLN13]

Parameters Setup

- $\mathcal{M} = R/tR = \mathbb{Z}_t[x]/(x^n+1)$, $R = \mathbb{Z}[x]/(x^n+1)$
- d power of 2, $q \in \mathbb{N}$ prime integer, $t \in \mathbb{N}$ s.t. $1 < t < q$
- $\chi_{\text{key}}, \chi_{\text{err}}$ distributions on R
- operations on ciphertexts in $R_q := \mathbb{Z}_q[x]/(x^n+1)$

The [BLLN13] SHE scheme (informal)

KeyGen(λ) :

- $[\cdots]$
- set sk $:= f \in R_q$

Encrypt(pk, m):

- for message $m + tR$, let $[m]_t$ be its representative
- sample $s, e \leftarrow \chi_{\text{err}}$
- output ciphertext $c = [\lfloor q/t \rfloor [m]_t + e + hs]_q \in R_q$

Decrypt(sk, c):

- output $m = \left[\left\lfloor \frac{t}{q} \cdot [fc]_q \right\rceil \right]_t \in R_t$

Key Recovery Attack against [BLLN13]

KeyGen(λ) :

- $[\cdots]$
- set sk $:= f(x) = f_0 + f_1 x + f_2 x^2 + \cdots + f_{n-1}x^{n-1} \in R_q = \frac{\mathbb{Z}_q[x]}{(x^n+1)}$

Encrypt(pk, m):

- $[\cdots]$
- output $c(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_{n-1}x^{n-1} \in R_q = \frac{\mathbb{Z}_q[x]}{(x^n+1)}$

Decrypt(sk, c):

- output $m = \left[\left\lfloor \frac{t}{q} \cdot [fc]_q \right\rceil\right]_t \in R_t$

Comparison with [DGM15]

- attack already exists in [DGM15], but require $6(t^2+t) < q$ and $B^2 < \dfrac{q}{36t^2}$ (conditions not assumed in [LATV12])
- our attack: works for all parameters. More efficient than [DGM15]:

	Our Attack	Attack from [DGM15]
(t is odd)	$\lceil \log_2(B/t) \rceil$	$n \cdot \lceil \log_2 B \rceil$
(t is even but not 2)	$\lceil \log_2(B/t) \rceil + n$	$n \cdot \lceil \log_2 B \rceil$
($t = 2$)	$\lceil \log_2(B/t) \rceil + n$	$n \cdot \lceil \log_2 B \rceil + n$

- n: power of 2; $B \ll q$ bound on coefficient of χ; $t \geq 2$ integer

Key Recovery Attack against [BLLN13]

KeyGen(λ) :

- $[\cdots]$
- set sk $:= f(x) = f_0 + f_1 x + f_2 x^2 + \cdots + f_{n-1} x^{n-1} \in R_q = \frac{\mathbb{Z}_q[x]}{(x^n+1)}$

Encrypt(pk, m):

- $[\cdots]$
- output $c(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_{n-1} x^{n-1} \in R_q = \frac{\mathbb{Z}_q[x]}{(x^n+1)}$

Decrypt(sk, c):

- output $m = \left[\left\lfloor \frac{t}{q} \cdot [fc]_q \right\rceil \right]_t \in R_t$

Key recovery attack - The main idea - we omit the details

- General idea: as usual, gradually reducing the interval in which the sk lie
- However, more complicated since we have to take into account and create several cases according to t odd, t even but $\neq 2$, and $t = 2$
- After each step k, f_i is determined up to an error $\frac{q}{2^k t}$
- we continue in this fashion until $\frac{q}{2^k t} \leq 1$

Conclusion and Future Directions

- SHE schemes from [BV11b, BV11a, BGV12, Bra12, GSW13, LATV12, BLLN13] suffer from key recovery attacks when the attacker is given access to the decryption oracle
- together with results from [LMSV12]: most existing SHE schemes suffer from key recovery attacks; not IND-CCA1 secure
- next step: to investigate whether it is possible to enhance these SHE schemes to avoid key recovery attacks and make them IND-CCA1 secure

Thank you for your attention!

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